Measuring Global Flow of Funds and Financial Network: Shock Dynamics and Propagation

Nan Zhang
Hiroshima Shudo University

Abstract

This study seeks to construct the Global Flow of Funds (GFF) matrix model to measure global financial stability. We use GFF data and the sectoral account data establish the sectoral from-whom-to-whom financial stock matrix (SFSM). The SFSM focuses on counterparty national and cross-border exposures of the sectors between China and the United States to construct country-specific financial networks and connect each country-level network based on cross-border exposures. We use financial network analysis to run an empirical analysis of SFSM to study the local propagation dynamics of quantity shocks in investment and financing.

Keywords: Sectoral data, who-to-whom matrices, inverse of Leontief, cross-border exposures, financial networks, shock propagation

1. Financial Network in the SFSM

The GFF data should be based on existing statistical data and therefore share many similarities of approach with them (IMF, 2006), such as enhanced consistency between International Investment Position (IIP), Coordinated Portfolio Investment Survey (CPIS), Coordinated Direct Investment Survey (CDIS), Bank for International Settlements (BIS), and Financial Accounts (FA), Balance of Payments (BOP), and Rest of the World (ROW) financial instruments.

There are two methods by which to compile the sector-by-sector matrix. One is to infer the debt ratio of a transaction item between sectors (Zhang, 2020); it is like a simple pro-rata method, and the other is to calculate the financial inflow–outflow based on the input–output principle. The latter is more direct and simpler, so this study adopts the second method. A deeper look demonstrates that exposures at the country and cross-border levels have increased in all governments. However, the decline in loan exposures was much larger at the cross-border level than it was at the country level, where the decline in equity exposures was more accentuated at the country level than it was at the cross-border level (Luiza, 2015). Precedents can also be seen in the preparation of the US–East Asia Financial Input–Output Table (Hagino et al., 2019). To observe the bilateral exposures at the country and cross-border levels and link them to the GFFM, we combine sectoral accounts data with data from the CDIS, CPIS, IIP, and BIS to calculate bilateral exposures between financial and nonfinancial sectors in three different financial instruments within and across CN, JP, US, and UK. For this purpose, it is necessary to create a counterparty country SFSM to convert the FBS prepared above into an SFSM.

To convert FBS to SFSM, first, we separate the assets and liabilities of each sector from the double-entry accounting-FBS and prepare the assets table (Table E) of each sector and the liability table (Table R) of each sector (Zhang, 2020, 95).

E is the financial asset matrix, and R is the financial liability matrix, based on Tables 1-4; t^E is the aggregate of financial instruments held by each sector, the row on the asset side, and t^R is the aggregate of financial instruments held by each sector, the row on the liability side. Here it is established that $t^E = t^R$. ε_j is the net financial liability of the *j*th sector, ρ_j is the net financial assets of the *j*th sector, and *t* is the total of assets or liabilities of each sector column. Each part of Tables E and R is represented as a matrix, and each element of E and R matrices is shown by e_{ij} and r_{ij}, respectively. Next, we can calculate the liability coefficient b_{ij} and asset coefficient d_{ij} using e_{ij} and r_{ij}, then deduce the capital inflow coefficient of Table Y (sector × sector) [1], and finally obtain Tables 1-4 [2].

This gives practical significance to the compilation and analysis of Table Y. In addition, the analysis of the ripple effect of financial risk at a certain point in time is essential. In addition, using Tables 1-4, we compiled Table Y, that is, the sector-by-sector SFSM. Tables 1-4 show the sector-by-sector SFSM of G-4 at the end of 2021, using data sources based on international standards, the same calculation method, the same currency unit, and five sectors for every country; thus, the outputs are internationally comparable. The rows in the table represent assets, and the columns represent liabilities. The row of each sector shows its stock of assets used in other sectors. Viewed from each sector column, the table can show the stock of the sector's financing from other sectors, and it can also show the fund operations within each sector (diagonal elements in the matrix). If the assets of a sector are greater than the liabilities, the net financial income of the sector is calculated as the Net assets in the row. If the assets of a sector are less than the liabilities, the net loss of the sectors is included as Net liabilities in the column. Through these four SFSM tables, we can know the basic structure of domestic and external assets and liabilities in CN, JP, US, and UK.

The compilation of the counterparty country sectoral SFSM reveals the W-t-W relationship between the Financial corporation (FC), Non-financial corporation (NFC), General government (GG), Households and HPISH [3] (HH), and ROW sectors, but this section focuses on the trading relationship between ROW in the SFSM and FC, NFC, GG, and HH in other countries' SFSM. Therefore, when determining the financial transactions or debt and creditor relationships between domestic sectors and overseas, especially the specific sectors of the counterparty, it is necessary to be sure of the basic data of the specific sectors of the counterparty. Therefore, there should be a basic dataset not only reflecting the FC, NFC, GG, and HH but also conforming to international uniform standards. To meet this requirement, two methods can be used: one is to integrate the existing data, and the other is to use the ratio for calculation.

We calculate the debt–bond relationship between counterparty sectors by directly utilizing the W-t-W information in their source data. Moreover, this method is used to identify links between each sector's outstanding amount of assets and each debt transaction item under the four sectors. Therefore, combining this method with the SFSM calculated above, we use the following three types of methods to prepare the bilateral SFSM.

First, from the perspective of the nature of financial commodities, the relationship between the asset-holding and liability-issuing sectors is very clear. For example, deposits and loans are issued and held by financial institutions. Second, we use financial instruments, the owners of which can be identified from other sources. For example, foreign deposits held by the government can be determined from GG; FDI is usually carried out by NFC; insurance pension, standardized guarantees, and investment trusts are usually held by HH; and financial derivatives are mainly issued and held by FC. Third, for some cases, such as treasury and financial bonds, where it is impossible to specifically distinguish the counterparty, it is calculated by the pro-rata approach.

Using this idea, we determine the following data sources and estimation methods for the sectors of the counterparty country. The following is an example of a methodology for determining the ratio of a country's ROW sector to another sector of a counterparty. Claims of sector i in Country A by sector j in Country B are calculated by multiplying Country A's foreign claims (ROW liabilities in Country A) by the share of Country B in Country A's foreign claims, the share of sector i's holdings of foreign assets in Country A, and the share of sector j's liabilities held by nonresidents in Country B. The data for

---

[1] For the calculation method of Table Y refer to Zhang (2020), 108-110.

[2] Tables 1-4 have been omitted due to space limitations

[3] HPISH stands for Non-Profit Institutions Serving Households

calculating the claims of sector i in JP by sector j in CN through the ROW sector are CPIS, CDIS, LBS, and IIP.

$$FC^{A}\_CN \rightarrow GG\_US = ROW^{L}_{CN} \times S_{CN \rightarrow US} \times S^{A}_{FC\_CN} \times S^{L}_{GG\_US}\,, \tag{1}$$

Where ROW^{L}_{CN} is the amount of Chinese ROW sector's liabilities (that is, the assets of CN) in debt securities coming from the sectoral accounts data; thus, LBS[4] data should be used. However, when estimating financial assets in the NFC sector, CDIS data should be used. $S_{CN \rightarrow US}$ is the share of the U.S. in Chinses foreign debt security claims coming from the CPIS[5]. $S^{A}_{FC\_CN}$ is the FC's share in the holdings of foreign debt securities in CN according to the IIP. And $S^{L}_{ROW\_US}$ is the GG's share in the U.S. liabilities in debt securities held by nonresidents according to the IIP dataset. Notably, sectoral accounts data, CPIS data, and IIP data are conceptually consistent among themselves in the sense, and their external claims compiled by country and instrument are almost equal.

Table 5 International SFSM with sectoral data (at the end of 2021, USD bn.)

Liablities Assets	FC_JP	NFC_JP	GG_JP	HH_JP	FC_CN	NFC_CN	GG_CN	HH_CN	FC_US	NFC_US	GG_US	HH_US	FC_UK	NFC_UK	GG_UK	HH_UK	ROW
FC_JP	16944	8120	8807	2793	11	30	3	0	1389	1025	405	80	410	33	20	8	2561
NFC_JP	4989	3989	536	225	1	146	0	0	264	952	118	23	149	189	23	9	45
GG_JP	2115	1538	1489	61	0	0	0	0	47	53	21	4	20	5	3	1	1021
HH_JP	15130	1783	271	31	3	7	1	0	151	171	68	13	17	4	3	1	476
FC_CN	27	29	18	5	22238	26865	9236	12316	590	639	253	50	137	32	19	7	2884
NFC_CN	14	17	9	3	16646	7193	137	31	179	280	80	16	387	107	60	23	765
GG_CN	0	0	0	0	6047	364	4	3	3	3	1	0	0	0	0	0	20
HH_CN	2	3	2	0	27159	3892	1722	92	34	38	15	3	20	5	3	1	304
FC_US	540	339	209	64	199	153	16	0	44623	43414	21036	14974	2022	176	105	40	8111
NFC_US	234	369	154	47	32	203	9	0	12553	18069	1687	383	59	1169	4	0	175
GG_US	0	0	0	0	0	0	0	0	2758	2260	1283	1210	0	0	0	0	567
HH_US	457	488	301	92	9	24	3	0	67362	34396	6063	671	17	4	3	1	9128
FC_UK	125	133	82	25	193	190	20	0	2014	1312	519	102	17224	4200	2671	1939	5023
NFC_UK	0	13	0	0	142	395	41	0	183	833	82	16	1700	835	129	180	5
GG_UK	0	0	0	0	3	9	1	0	37	42	17	3	415	226	130	112	189
HH_UK	10	11	7	2	21	56	6	0	158	178	71	14	7853	1392	80	83	692
ROW	655	805	579	178	676	2,245	269	1	10715	12320	5407	1068	5458	533	1103	421	

Source: Tables 5 from Dataset with 720 financial balance sheets of OECD Statistics; CPIS's Table 5, CDIS's Table 3, and IIP's Table 5 that are published by IMF; LBS's Table A6.2 of BIS.

---

[4] To avoid double counting, the claims, that is, loans and deposits of CN to US in Table A6.2-S banks' cross-border positions on residents of CN in the LBS account are subtracted from the claims of FC by ROW in SFSM.

[5] CPIS: Table 5, Reported Portfolio Investment Assets by Sector of Holder, and Sector and Economy of Nonresident Issuer for Specified Economies, December 2018.

Using Eq. (1) and relevant data, we compile an international SFSM with counterparty country-sectors, as shown in Table 5. For it to be consistent with the creditors and debtors in Tables 1-4, we have defined the rows and columns in Table 5, the columns in Table 5 represent liabilities, and the rows represent assets[6]. Table 1-4 are W-t-W tables showing the credit relationship between the assets and liabilities of domestic counterparties sectors for each country, but Table 5 provides more detailed sector-to-sector information among the four countries than Table 1-4 does. Therefore, a column categorizes a sector's assets by counterparty, so by observing the columns, we can know both the use of financial assets among domestic sectors and the creditor's rights held by various sectors of various countries and cross-border sectors of other countries. The ROW in the bottom row of Table 5 refers to financial investments (creditors) by counterparty country sectors in countries other than the target country. The total assets of the ROW sector are calculated by summing up the total assets of the ROW sector in all the G-4 economies. A row categorizes a sector's liabilities by counterparty, so from the perspective of the rows, we can observe not only the financial liabilities between domestic sectors but also the liabilities held by counterparty countries and cross-border sectors.

Table 5 uses the debt and claims relationship between the domestic sectors of CN, JP, the US, and the UK at the end of 2021 and the bilateral risk exposure of one country to the other to construct the financial network of a specific country. It describes how sectoral account data can be harmonized with CDIS, CPIS, LBS, and IIP data to derive information regarding cross-border risk exposure from inside to outside at each country level. The columns of Table 5a are set as liabilities and rows as assets, which belongs to the matrix table of Stone-mode, focusing on observing the situation and effect of financing counterparties of various sectors.

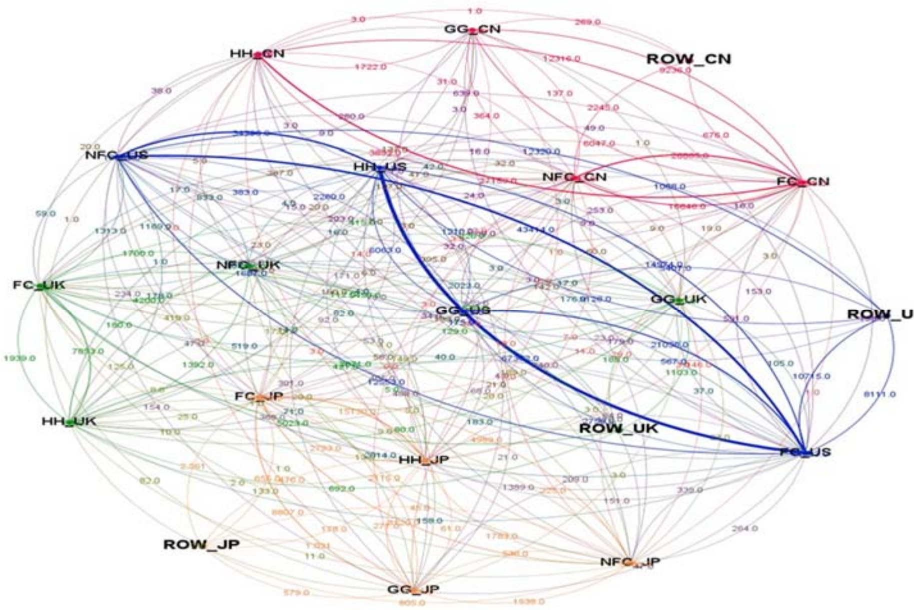

Figure 1 Assets and liabilities network in the sectors of G-4 at the end-2021

The W-t-W data of the GFF can be seen as a network of interrelationships in which the nodes—the elements interlinked in the network—are sectors. There are three important key points.

Node is the positions of the nodes is arbitrary, but their sizes are proportional to the countries' holdings of liabilities of a given type, to facilitate the identification of systemically important countries. Edge is the links between nodes—are asset/liability links. The edges in the network are "weighted" by the amounts involved in every asset or liability relationship. And Link is the width of

---

[6] This is the arrangement of rows and columns which designed by Stone's formula.

the link is also proportional to the size of each sector's exposure to another sector. Since networks are constructed to assess financial stability, its width is based on the creditor sector’s capacity to absorb the potential loss represented by this claim. Consider SFSM (Table 5) as a financial network diagram shown in Figure 1.

Empirical research of network theory mostly adopts the following methods: a relationship model between multiple nodes is constructed using balance sheet information. Then, the stability of the network is tested by simulating the impact of a shock. Because of space constraints, we focus on degree centrality in the network analysis to illustrate the importance and influence on sectors of G-4 countries in the GFF network.

2. Interpreting Financial Networks in the sectors of G-4

In this section, we will develop centrality measures on C, which directly represents the net of interlinks, in particular, eigenvector centrality, capturing direct and indirect links.

Based on the GFFM model created by our previous research (Zhang and Zhao, 2019), bilateral exposures across N countries in a financial instrument k can be expressed in a n x n matrix in which the element y_{ij} denotes a claim of country i vis-à-vis country j. So, the sum of each column j denotes the aggregate holdings of assets of country j in instrument k ($a_{j,k}$), and the sum of each row i denotes the aggregate holdings of liabilities of country i in instrument k ($l_{i,k}$). Aggregate assets ($a_{j,k}$) and liabilities ($l_{i,k}$) per country are observed, but bilateral exposures need to be estimated.

$$Y_k = \begin{pmatrix} y_{11} & \cdots & y_{1n} \\ \cdot & \cdot & \cdot \\ y_{n1} & \cdots & y_{nn} \end{pmatrix} \text{ with } \sum_{i=1}^{n} y_{ij} = a_{j,k} \text{ and } \sum_{j=1}^{n} y_{i,j} = l_{i,k}$$

In order to represent how the investment of the various countries react to the investment of others (in order to finance them), we set Δs as an exogenous variable which the shock itself, indicating the original changes in investment, its transpose vector can be represented as follows.

$$\Delta s = (0, \cdots, -s, 0, \cdots, 0)' \qquad (2)$$

By the W-t-W framework, a matrix algebra presentation of GFFM can be shown by $T = Y + \Delta s$. Where T is the vector $T = \begin{pmatrix} t_1 \\ \cdot \\ t_n \end{pmatrix}$.

Let us now define the elements c_{ij} is the ratio of funds raised from country i to the total external financing of country j, that is, $c_{ij} = \frac{y_{ij}}{t_j}$, the investment ration matrix is shown in C. Where C is the matrix of c_{ij} determined by the form of the n × n order, and can get $y_{ij} = c_{ij} * t_j$, and we can get the diffusion matrix represented as follows. $T = C * T + \Delta s$. Where $T_k = C * \Delta s$, (k=0, 1, 2, … , n), when k=o that called a direct effect, k=1 that is called an indirect first-order effect, k=2 that is indirect second-order effect,… , and k=n that is indirect n-order effect, as shown in the following equation.

$$\begin{aligned}\xi_k &= T_0 + T_1 + T_2 + \cdots + T_k \\ &= T_0 + CT_0 + C^2T_0 + \cdots + C^kT_0 \\ &= (I + C + C^2 + \cdots + C^k)T_0\end{aligned} \qquad (3)$$

And when $k \to \infty$, $\xi_\infty = (I-C)^{-1}T_0$ (4)

Formula 5 reflects the limiting effect of n-order.

Where $(I-C)^{-1}$ is the inverse of Leontief. Whereas Leontief deals with input per unit of output, we consider here financing per unit of investment, but the overall logic behind the two representations is the same one. The elements in the diffusion matrix in our model have interesting interpretations in terms of well-known financial ratios. Thus, $c_{1,j}$ and $c_{2,j}$ are the ratios of financing from a country to other country, respectively, to the investment of the country (also assuming that, i.e. when $i = j$, $C_{i,j} = o$, that is, excluding the domestic portfolio investment of the country). The ratios $c_{1,j}$, $c_{2,j}$, $c_{3,j}$ and $c_{i,j}$ represent the mix of financing sources for portfolio investment, indicating how a country resort to other countries funding, usually treasuries and financial bonds.

2.1. Shock dynamics of the sectors of G-4

This section uses the shock dynamics model (see formula 3) to measure the impact of investment changes in ROW_CN in G-4 on other sector, with a focus on JP, CN, US, and UK. In addition, according to the changes in the financial investment of JP, CN, US, and UK, with regard to the above vector Δ s, the shock expressed by the unitary term is as follows:

$$\Delta s = (1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; -1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1)' \qquad (5)$$

That is, it is assumed that the portfolio investment of the United States will be reduced by 1 unit, the portfolio investment of Japan and China will be increased by 1 unit, and the portfolio investment of other countries will be assumed unchanged with an increment by 0. Therefore, according to formula (4), using Δ s and investment ration matrix C, we can present the decomposition into the first 15 orders in G-4's case which show in Table 6 and speculate the shock of the changes in the investment on CN, JP, US, and UK.

The shock effects shown in Table 6 can be decomposed into four parts that (i) the shock itself - the vector Δs indicating the original changes in investment, (ii) the investment effort needed to finance such original investment change which the vector $T_0 = C * \Delta s$, (iii) the investment effort needed to finance the original investment change which the vector $C^2T_0, C^3T_0, \cdots C^{15}T_0$, and (iv) so on into infinite n-order investment efforts which the vector $(I-C)^{-1}\Delta s$.

In Table 6, the changes in investment and financing triggered by shocks are governed by the set of direct and indirect relationships embedded in the w-t-w diffusion matrix, including intricate investment/ financing paths of any order, even beyond the fifteen-order one referred to above for our example. Next, we propose a decomposition of the shocks on the United States, Japan, and China that separates these individual n-order effects.

We plotted Figure 2-5[7] using the observations in Table 6, which show the shock effects in FC_CN, GG_CN, FC_US, and GG_US. The shock itself, or the first-order effect, consists of a reduction in external fund inflow in CN and financial liability increase in CN, JP, the US and the UK.

---

[7] Figure 2-6 have been omitted due to space limitations.

Since FC_US has the largest market share of financing in the world, even assuming that increase in original financing by one unit, its direct effect will be shown with 0.6584, but it quickly recovers to 0.1359 of its original position after the six-order indirect effects, then the shock effect declines gradually after the 9th-order, tightening to zero effect by the 15th-order. From a cumulative effect perspective, it exhibits a significant impact function, ranging from 2.1341 for the second order to 3.2116 for the eighth order. Subsequently, the positive shock started to diminish but gradually climbed to 3.4155 at the fifteenth order (refer to Figure 8). The overall limit value for FC_US, encompassing both primary and indirect effects, stands at 3.4516 (see Table 6).

Table 6 Shock dynamics for SFSM by Table 5 (Stone-model)

Sectors	Δs	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	· · ·	$(I-C)^{-1}*\Delta s$
FC_JP	**1.0000**	1.0429	0.7921	0.5740	0.4311	0.3238	0.2436	0.1833	0.1380	0.1040	0.0783	0.0591	0.0445	0.0336	0.0254	0.0192		5.1528
NFC_JP	0.0000	0.1340	0.1801	0.1500	0.1140	0.0859	0.0646	0.0487	0.0367	0.0276	0.0208	0.0157	0.0119	0.0090	0.0068	0.0051		0.9269
GG_JP	0.0000	0.2993	0.1056	0.0798	0.0600	0.0453	0.0340	0.0256	0.0193	0.0145	0.0109	0.0082	0.0062	0.0047	0.0035	0.0027		0.7281
HH_JP	0.0000	0.4724	0.3943	0.3082	0.2254	0.1695	0.1273	0.0958	0.0721	0.0543	0.0409	0.0308	0.0232	0.0175	0.0132	0.0100		2.0860
ROW_JP	**1.0000**	0.0154	0.0407	0.0292	0.0224	0.0168	0.0127	0.0095	0.0072	0.0054	0.0041	0.0031	0.0023	0.0017	0.0013	0.0010		1.1757
FC_CN	**1.0000**	-0.4220	0.0843	-0.0873	0.0012	-0.0160	0.0016	0.0020	0.0062	0.0070	0.0078	0.0078	0.0075	0.0070	0.0064	0.0057		0.6540
NFC_CN	0.0000	0.0406	-0.0670	0.0183	-0.0076	0.0059	0.0027	0.0049	0.0045	0.0046	0.0043	0.0039	0.0036	0.0032	0.0028	0.0024		0.0409
GG_CN	0.0000	0.0752	-0.0334	0.0062	-0.0068	0.0000	-0.0012	0.0002	0.0002	0.0005	0.0006	0.0007	0.0007	0.0006	0.0006	0.0005		0.0480
HH_CN	0.0000	0.2848	-0.1345	0.0196	-0.0285	-0.0009	-0.0049	0.0009	0.0014	0.0029	0.0032	0.0034	0.0034	0.0032	0.0030	0.0027		0.1767
ROW_CN	**-1.0000**	0.0090	0.0002	-0.0036	0.0003	-0.0006	0.0002	0.0001	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002		-0.9908
FC_US	1.0000	0.6584	0.4757	0.3343	0.2467	0.1824	0.1359	0.1017	0.0764	0.0576	0.0436	0.0330	0.0251	0.0191	0.0145	0.0111		3.4516
NFC_US	0.0000	0.1012	0.1005	0.0807	0.0597	0.0445	0.0332	0.0249	0.0187	0.0141	0.0107	0.0081	0.0062	0.0047	0.0036	0.0027		0.5225
GG_US	0.0000	0.0385	0.0255	0.0175	0.0127	0.0093	0.0069	0.0051	0.0038	0.0029	0.0022	0.0016	0.0012	0.0009	0.0007	0.0005		0.1311
HH_US	0.0000	0.7915	0.3991	0.3012	0.2166	0.1598	0.1184	0.0882	0.0660	0.0496	0.0374	0.0283	0.0214	0.0162	0.0124	0.0094		2.3460
ROW_US	**1.0000**	0.0749	0.0728	0.0536	0.0389	0.0286	0.0212	0.0158	0.0118	0.0089	0.0067	0.0050	0.0038	0.0029	0.0022	0.0017		1.3542
FC_UK	**1.0000**	1.1677	0.7835	0.6225	0.4753	0.3664	0.2816	0.2165	0.1664	0.1279	0.0983	0.0755	0.0580	0.0446	0.0343	0.0263		5.6328
NFC_UK	0.0000	0.0512	0.0683	0.0501	0.0395	0.0302	0.0233	0.0179	0.0138	0.0106	0.0082	0.0063	0.0049	0.0037	0.0029	0.0022		0.3408
GG_UK	0.0000	0.0370	0.0233	0.0197	0.0149	0.0115	0.0088	0.0068	0.0052	0.0040	0.0031	0.0024	0.0018	0.0014	0.0011	0.0008		0.1447
HH_UK	0.0000	0.3125	0.2822	0.2044	0.1598	0.1225	0.0943	0.0725	0.0558	0.0429	0.0329	0.0253	0.0195	0.0150	0.0115	0.0088		1.4894
ROW_UK	1.0000	0.1521	0.2024	0.1403	0.1108	0.0848	0.0653	0.0502	0.0386	0.0297	0.0228	0.0175	0.0135	0.0104	0.0080	0.0061		1.9728

As shown in Figure 3-4, under the influence of positive shocks in FC_US and constraints from other sectors in China, Japan, and the UK, GG_US experiences relatively minor positive shock effects. The first-order effect, i.e., the direct effect, is 0.0385, after then decreases to 0.0038or the eighth-order positive effect and gradually converges to zero thereafter. The comprehensive limit effect is 0.1311 (see Table 6), which is lower than the shock effects on the government sectors of China, Japan, and the UK.

When we assume that FC_CN increases by 1 and ROW_CN decreases by -1, while both FC sector and ROW sector in JP, the US and the UK increase by +1 unit, and the change of other sectors is 0, the change of FC_CN is shown in Figure 10. The direct effect of FC_CN is -0.422, and its negative effect changes from the first-order to the 6th-order to a lower positive effect of 0.0016. This low positive effect continues until 15th-order, with a limit effect of 0.654. Moreover, the cumulative effect of hedging negative and positive effects in order 15 is 0.619, which is significantly lower than that of the FC sector in JP, the US, and the UK (see Table 6). It can be seen that FC_CN is less able to cope with debt shocks because of the decrease in overseas financing.

2.2. Shock propagation in the SFSM

To focus on the impact of changes in sectors of G-4, we adapt the 20-order matrix in Table 5 to the 20-order matrix C (see Eq.3), as shown in Table 8[8]. And in order to reflect the shock of a sector's investment changes on other sectors, the diffusion matrix C as an operator on the vector can be calculated as follows.

We put *V*, λ *as* the matrix of eigenvectors and diagonal matrix of eigenvalues of C -which we assume as diagonalizable, as in our example- respectively, so that $C = V * \lambda * V^{-1}$ and $C^n = V * \lambda^n * V^{-1}$ (see Meyer, 2000). This allows the following representation of the n-effects:

$$C^{n-1} * \Delta s = V * \lambda^{n-1} * (V^{-1} * \Delta s) \tag{6}$$

The vector $(V^{-1} * \Delta s) = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ \vdots \\ e_{20} \end{pmatrix}$ contains the components of the shock vector Δs expressed in the base of eigenvectors, where Δ s (see Eq.6). That is, it is assumed that the ROW_CN will be reduced by 1 unit, the ROW_JP, ROW_US, and ROW_UK will be increased by 1 unit, and other sectors of G-4 will be assumed unchanged with an increment by 0.

The matrix of eigenvectors V and the diagonal matrix λ of eigenvalues can set as below.

$V = \begin{pmatrix} v_{1,1} & v_{1,2} & v_{1,3} & v_{1,4} & \cdots & v_{1,20} \\ v_{2,1} & v_{2,2} & v_{2,3} & v_{2,4} & \cdots & v_{2,20} \\ v_{3,1} & v_{3,2} & v_{3,3} & v_{3,4} & \cdots & v_{3,20} \\ v_{4,1} & v_{4,2} & v_{4,3} & v_{4,4} & \cdots & v_{4,20} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ v_{20,1} & v_{20,2} & v_{20,3} & v_{20,4} & \cdots & v_{20,20} \end{pmatrix}$ and $\lambda = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_{20} \end{pmatrix}$ in our 20×20 example, (6) can also be expressed as follows:

$$C^{n-1} * \Delta s = \lambda_1^{n-1} * e_1 * \begin{pmatrix} v_{1,1} \\ v_{2,1} \\ v_{3,1} \\ v_{4,1} \\ v_{\vdots,1} \\ v_{20,1} \end{pmatrix} + \lambda_2^{n-1} * e_2 * \begin{pmatrix} v_{1,2} \\ v_{2,2} \\ v_{3,2} \\ v_{4,2} \\ v_{\vdots,2} \\ v_{20,2} \end{pmatrix} + \lambda_3^{n-1} * e_3 * \begin{pmatrix} v_{1,3} \\ v_{2,3} \\ v_{3,3} \\ v_{4,3} \\ v_{\vdots,3} \\ v_{20,3} \end{pmatrix} + \lambda_{\dots}^{n-1} * e_{\dots} * \begin{pmatrix} v_{1,20} \\ v_{2,20} \\ v_{3,20} \\ v_{4,20} \\ v_{\vdots,20} \\ v_{20,20} \end{pmatrix} \tag{7}$$

The individual n-order effects being expressed as a linear combination of the eigenvectors of the

---

[8] Table 8 is not included in this document due to space constraints.

diffusion matrix, use the 20-order diffusion matrix C, we can get the corresponding eigenvalues λ with the components of the shock in the base formed by the eigenvectors V. The transpose eigenvalues λ denoted as λ `, which shown as below.

λ= (	0.8210	0.7638	0.7304	0.6008	-0.3814	-0.1793	-0.1440	-0.1220	0.0921	0.0921	0.1078	0.0914	-0.0527	0.0364	-0.0289	-0.0288	-0.0227	-0.0227	-0.0214	0.0021	)`

The matrix of eigenvectors V and Inverse matrix of eigenvector V^{-1} are put at the end of this paper as Annex Table 1 and Annex Table 2, and use $V^{-1} * \Delta s = E$ can get the vector E, the transpose of matrix E is denoted as E^T as following.

E^T= (	1.0428	0.1339	0.2993	0.4724	0.0154	-0.422	0.0407	0.0752	0.2848	0.009	0.6584	0.1011	0.0385	0.7915	0.0749	1.1678	0.0513	0.037	0.3124	0.1521	)

In this way, we can get the decomposition of the impact on ROW_CN when China's foreign asset utilization decreases, that is, the eigenvector decomposition of (n>1)-order effects for ROW_CN which shown as Table 9[9].

This presentation would allow us better understanding the features that govern the propagation effects and link them to network centrality, as well as perform dimensionality reduction to simplify the presentation of the shock dynamics. Figure 7-8 shows the decomposition of the effects on the ROW_CN and ROW_US for n>1 (indirect effects).

According to formula (6) and use these eigenvalues and eigenvectors which solved above, we can analyze and decompose the shock propagation on the United States, Japan, and China. The shock propagation equation can be made for each eigenvalue as below. This presentation would allow us better understanding the features that govern the propagation effects and link them to network centrality, as well as perform dimensionality reduction to simplify the presentation of the shock dynamics.

We first calculated the shock propagation on China by formula (7), Figure 7 shows the decomposition of the effects on China for n>1 (indirect effects). The power after the shock is decomposed into a persistent negative sub-effect (red line (λ1) in Figure 7), and three sign-oscillating sub-effects (purple (λ11), green (λ13), baby blue (λ2) inducing the alternation of positive and negative effects described. The nature of the signs as oscillating or not depends on the sign of the corresponding eigenvalue, those with negative value (0.821 in ROW_CN) delivering a constant sign contribution which depends on the sign of the product of the component of the shock in the eigenbase (-0.0313 in ROW_CN) and the sector component in the eigenvector associated to the eigenvalue (1.0428, delivering an overall positive sign path).

The size of the sub-effects depends on the corresponding module of the eigenvalues[10], the components of the eigenvectors and the components of the shock. Therefore, when the sub-effects linked to the eigenvalue are extremely small and disappearing fast for ROW_CN, to the extent that they can be totally ignored.

By the Fig. 8 we can know that the persistence of the n-order effects depends on the module of the eigenvalue. Thus, the sub-effects linked to the eigenvalue 0.7638 (orange (λ2) line in Figure 8), the eigenvalue 0.6009 (yellow (λ4) line in Figure 8), the eigenvalue 0.821 (blue (λ1) line in Figure 8), and

---

[9] Table 9 has been omitted due to space limitations.

[10] Eigenvalues, eigenvectors and shocks in the base of eigenvectors are in general complex numbers if we allow for diffusion matrices that are diagonalizable in the complex plane. This case pertains to economic analysis. When the eigenvalue or eigenvector exhibits a very small imaginary component, considering only the real part of the complex number does not significantly impact the accuracy of prediction. Therefore, this paper exclusively focuses on the real component of the complex number

the eigenvalue 0.7304 (gray (λ3) line in Figure 8) show the significant sub-effects before 6th-order, but after that, the associated sub-effects are extremely small and disappear quickly, to the extent that they can be totally ignored for characterizing the shock dynamics. In addition, the eigenvalue λ12, λ16, λ6 and λ14 have a short-term impact on the ROE_US, but the duration is short and tends to stop at the second-order. Therefore, we can know that the shock effect on the ROW_US by the NFC_JP, FC_JP, and GG_JP to the 6th-order, short-term effect on the ROE_US by the NFC_US, FC_UK, FC_CN, and HH_US, and then will no longer have a shock effect.

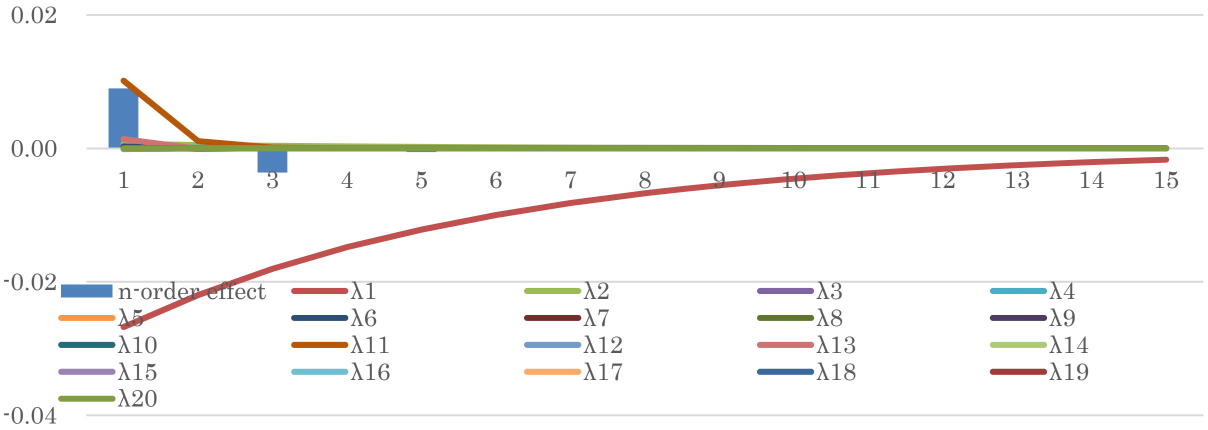

Figure 7. Shock propagation on ROW_CN

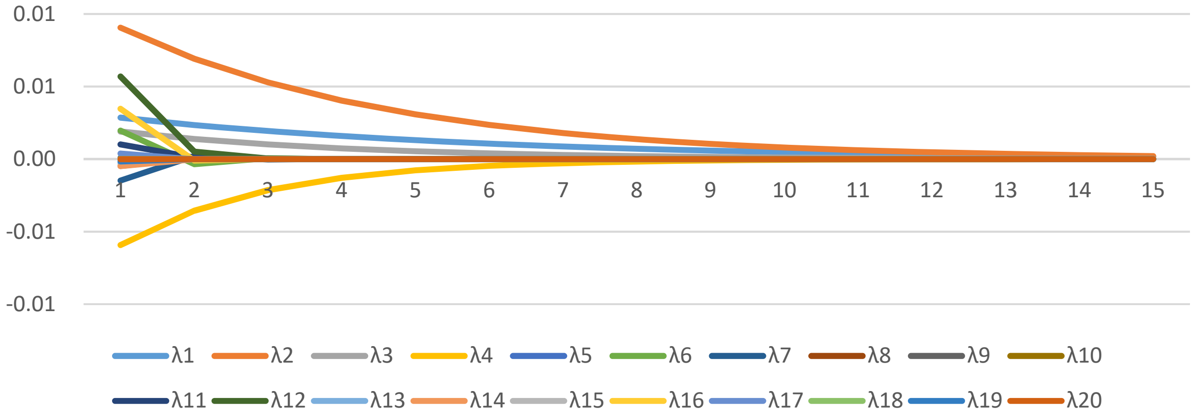

Figure 8. Shock propagation on ROW_US

References

Antoun de Almeida, L. (2015) A Network Analysis of Sectoral Accounts: Identifying Sectoral Interlinkages in G-4 Economies. IMF Working Paper WP/15/111, Washington DC

Giron, C., M. Rodriguez Vives and A. Matas (2018) Propagation of Quantity Shocks in Who-to-whom Networks. Paper prepared for the 35th IARIW General Conference

Nan Zhang and Xiuzhen Zhao(2019) Measuring Global Flow of Funds: A Case Study on China, Japan and the United States, Economic Systems Research, Vol. 31, No.4, 520-550.

Zhang, N. (2020) Flow of Funds Analysis: Innovation and Development, Springer, Cha.7-10.

Hagino, S., and J. Kim (2021) Compilation and analysis of international from-whom-to-whom financial stock table for Japan, Korea, the United States, and China, Journal of Economic Structures.

Zhang, N. (2022) Measuring Global Flow of Funds: Who-to-whom Matrix and Financial Network, Japanese Journal of Statistics and Data Science (2022) 5: 899-942.