$S_{x+c}^{2} = S_{x}^{2}$ を証明せよ。

$\because \quad x_1+c, x_2+c, x_3+c, \cdots x_n+c$ の平均値は

$$\overline{x}' = \frac{\sum_{i=1}^{n}(x_i+c)}{n} = \frac{\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}c}{n} = \frac{\sum_{i=1}^{n}x_i}{n} + \frac{\sum_{i=1}^{n}c}{n} = \overline{x} + \frac{nc}{n} = \overline{x} + c$$

となるので、

$$\therefore \quad S_{x+c}^{2} = \frac{\sum_{i=1}^{n}\left\{(x_i+c) - \overline{x}'\right\}^2}{n} = \frac{\sum_{i=1}^{n}\left\{(x_i+c) - (\overline{x}+c)\right\}^2}{n} = \frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n} = S_x^2$$

$S_{xc}^{2} = c^2 S_{x}^{2}$ を証明せよ

$\because \quad x_1c, x_2c, x_3c \cdots x_nc$ の平均値は

$$\overline{x}' = \frac{\sum_{i=1}^{n}x_ic}{n} = \frac{c\sum_{i=1}^{n}x_i}{n} = c\overline{x}$$

となるので

$$\therefore \quad S_{xc}^{2} = \frac{\sum_{i=1}^{n}(x_ic - \overline{x}')^2}{n} = \frac{\sum_{i=1}^{n}(x_ic - c\overline{x})^2}{n} = \frac{\sum_{i=1}^{n}c^2(x_i-\overline{x})^2}{n} = \frac{c^2\sum_{i=1}^{n}(x_i-\overline{x})^2}{n} = c^2S_x^2$$